SyDe312 Numerical Methods - Test 2

4 March 2005, 12:30-1:20

Read the questions CAREFULLY. You can easily make them much more difficult and much longer than intended!

1. (a) [2 marks] Derive the iteration formula (in matrix form) for the Gauss-Seidel method for finding an approximate solution to the system of linear equations $Ax = b$.

   (b) [1 mark] What is the iteration matrix G and what conclusions can you make about convergence based on its properties?

   (c) [4 marks] Find an approximate solution to the system $Ax = b$ using the Gauss-Siedel method for $A = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 5 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 33 \\ 26 \\ 30 \\ 15 \end{bmatrix}$. How many iterations are required to achieve 4 decimal place accuracy? [hint: use matlab to carry out the necessary arithmetic in the simplest way]

2. [8 marks] Consider the polynomial $p(x) = 1.0x^5 - 7.2x^4 + 17.0x^3 - 9.2x^2 - 16.8x + 17.6$.

   (a) Generate a plot of $p(x)$ in matlab to obtain a general view of where the roots lie and provide a sketch plot of $p(x)$ showing this.

   (b) Use your favourite root-finding method to find a value for the smallest root x_1 of $p(x)$. You should include a brief description of the method you used, the initial guess, some comments about the convergence rate, and any other information you consider approriate. You can (should) use matlab to implement the arithmetic necessary to generate the root. DO NOT use any of the built-in matlab methods for this. [hint: Don't waste time on fancy matlab programming. Just use the quickest and simplest way to get the desired result.]

   (c) Use polynomial deflation to eliminate the factor $x - x_1$ of $p(x)$ corresponding to the root you found in part (b) and obtain the polynomial $q(x)$ of degree 4 [hint: `deconv`].

   (d) Investigate the roots of $q(x)$ by generating plots in matlab for various ranges of x until you have a clear view. Provide a sketch plot of $q(x)$ showing this information.

   (e) Use your favourite root-finding method to locate another root, followed by a polynomial deflation step to remove the corresponding factor.

   (f) Repeat parts (d) and (e) until you have accurate values for ALL the roots of $p(x)$.

   (g) Summarize the results giving all the roots you found for $p(x)$ and updated sketch graph(s) as required to show clearly the behaviour of the function around the roots. [hint: you'll probably need two sketch graphs for clarity.]

   (h) Compare the roots you found to those obtained using a built-in matlab root-finding alorithm (e.g. `roots`). Which of the methods gives more accurate roots, yours or the built-in algorithm? Why?